Discussion 5B

CS 70, Summer 2024

1 Conditional Probability

The local weather channel just released a statistic for the months of November and December. It said:

- The probability that it would rain on a windy day is 0.3.
- The probability that it would rain on a non-windy day is 0.8.
- The probability of a day being windy is 0.2.

As a student in CS 70, you are curious to play around with these numbers. Find the probability that:

(a) A given day is both windy and rainy.

(b) A given day is rainy.

(c) For a given pair of days, exactly one of the two days is rainy. (You may assume that the weather on the first day does not affect the weather on the second.)

2 Random Variables

Let X and Y be random variables, each taking values in the set $\{0, 1, 2\}$, with joint distribution

$$\begin{array}{lll} \Pr[X=0,Y=0]=1/3 & \Pr[X=0,Y=1]=0 & \Pr[X=0,Y=2]=1/3 \\ \Pr[X=1,Y=0]=0 & \Pr[X=1,Y=1]=1/9 & \Pr[X=1,Y=2]=0 \\ \Pr[X=2,Y=0]=1/9 & \Pr[X=2,Y=1]=1/9 & \Pr[X=2,Y=2]=0. \end{array}$$

(a) What are the marginal distributions of X and Y?

(b) What is the conditional distribution of X conditioning on $Y = 0$?

(c) What is the conditional distribution of X conditioning on $1 \leq X + Y \leq 2$?

3 Mutually Independent Events

There are three mutually independent events A, B and C, with $\mathbb{P}(A) = 2/5, \mathbb{P}(B) = 3/5$ and $\mathbb{P}(C) = 3/10$. Calculate the following.

(a) $\Pr[A|B]$.

(b) $\Pr[A \cap B]$ and $\Pr[A \cup B]$.

(c) $\Pr[A \cap B \cap C]$ and $\Pr[A \cup B \cup C]$.

4 Working with Distributions

(a) Five fair coins are flipped and the random variable Y is defined as the number of tails observed. Find the distribution of Y.

(b) Suppose a fair six-sided die is rolled until a number smaller than 3 is observed. Let N be the total number of times the die is rolled. Find the distribution of N.