Discussion 4D

CS 70, Summer 2024

1   Urns and Marbles

Lance has two urns. Urn 1 has one black marble and three white marbles. Urn 2 has two black marbles and two white marbles.

Lance picks an urn at random and draws two marbles from the urn with replacement.

(a) Suppose that Lance sees both colors of marbles. Which of the two urns is more likely to have been chosen by Lance?

(b) Find the chance that Lance chose Urn 1 given that he sees both colors of marbles.

(c) Find the chance of B_1, the event that Lance draws a black marble on his first draw.

(d) Find the chance of W_2, the event that Lance draws a white marble on the second draw.

(e) Determine whether the events B_1 and W_2 are independent.

2 The Matching Problem

In a house of n cats, each cat has a designated bowl with their name on it. However, when it comes time for dinner, each cat selects a bowl at random such that no two cats select the same bowl. That is, the cats are permuted randomly to the bowls such that all permutations are equally likely.

(a) Find the chance that the first cat selects the right bowl.

(b) Find the chance that both of the first two cats select the right bowl.

(c) Find the chance that none of the cats select the right bowls.

3   Bag of Marbles

Timothe has a bag of n marbles, of which r are red, b are blue, and g are green. Timothe draws marbles from the bag without replacement.

(a) Find the chance that the first marble is red.

(b) Find the chance that the second marble is red.

(c) Find the chance that the tenth marble is red.

(d) Find the chance that the first marble is red given that the fourth marble is blue.

(e) Find the chance that exactly four of the first ten marbles are red.